Lecture III: The Thermal History

Julien Lesgourgues

EPFL & CERN

London, 14.05.2014

Thermal history

- treated by the module thermodynamics.c. So this lecture will refer mainly to the content of input/thermodynamics.h, source/thermodynamics.c, and to the structure referred as th:

```
struct thermodynamics th;
```

with fields th.blabla, or through the pointer pth:

```
struct thermodynamics * pth;
```

with fields pth->blabla.

Thermal history

- treated by the module `thermodynamics.c`. So this lecture will refer mainly to the content of `input/thermodynamics.h`, `source/thermodynamics.c`, and to the structure referred as `th`:

  ```
  struct thermodynamics th;
  ```

  with fields `th.blabla`, or through the pointer `pth`:

  ```
  struct thermodynamics * pth;
  ```

  with fields `pth->blabla`.

- the goal of this module is to solve the thermal history and store the results in a table. It should provide a function able to interpolate within this table at any value of redshift.

Thermal history

- treated by the module thermodynamics.c. So this lecture will refer mainly to the content of input/thermodynamics.h, source/thermodynamics.c, and to the structure referred as th:

```
struct thermodynamics th;
```

with fields th.blabla, or through the pointer pth:

```
struct thermodynamics * pth;
```

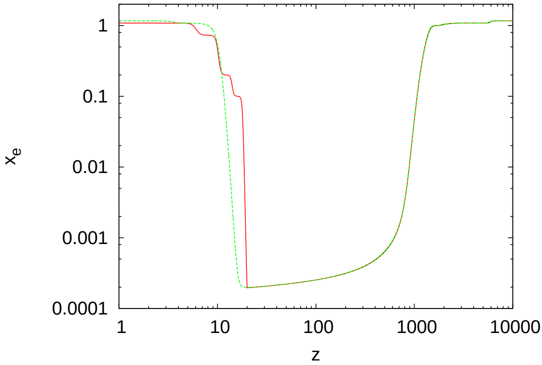
with fields pth->blabla.

- the goal of this module is to solve the thermal history and store the results in a table. It should provide a function able to interpolate within this table at any value of redshift.
- other modules should be able to know all useful thermodynamical quantities at any given redshift.

Thermodynamics

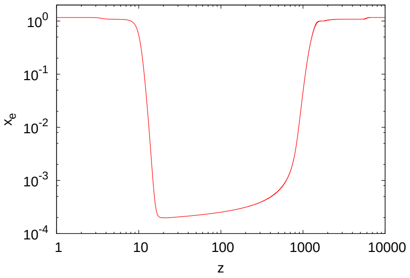
In this lecture we will address the following questions:

- what is assumed in CLASS about recombination and reionisation?
- how are they implemented?
- how to prepare plots of thermodynamical quantitites.



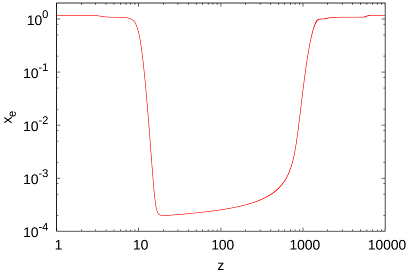
Thermal history

- free electron fraction $x_e = n_{\text{free electrons}}/n_p$ (not counting protons in He nuclei, so x_e can be > 1)



Thermal history

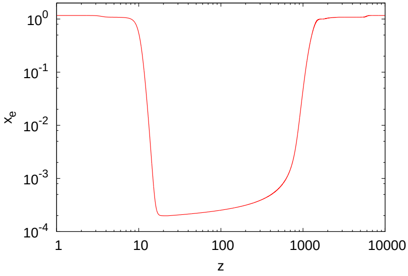
- free electron fraction $x_e = n_{\rm free\ electrons}/n_p$ (not counting protons in He nuclei, so x_e can be > 1)



- observe the first/second He recombination, the H recombination (around $T \sim 0.3$ eV, not 13.6 eV!), the H reionisation, the first He reionisation

Thermal history

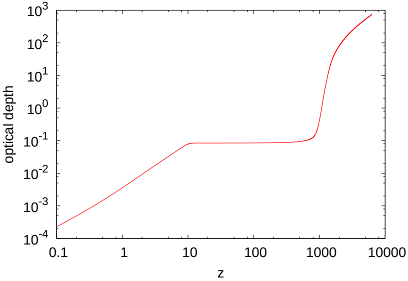
- free electron fraction $x_e = n_{\text{free electrons}}/n_p$ (not counting protons in He nuclei, so x_e can be > 1)



- observe the first/second He recombination, the H recombination (around $T \sim 0.3$ eV, not 13.6 eV!), the H reionisation, the first He reionisation
- Thomson scattering rate $\kappa' = \sigma_T a n_p x_e$: universe becomes transparent when $\kappa' < H$, i.e. at recombination

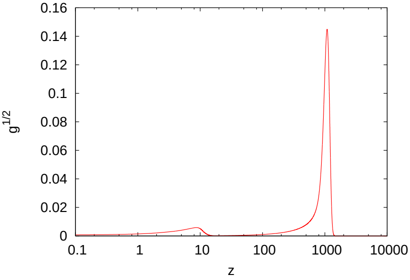
Thermal history

- optical depth $\kappa(\tau) = \int_{\tau}^{\tau_0} \kappa' d\tau$ = depth of the cosmic fog



Thermal history

- visibility function $g(\tau) = \kappa' e^{-\kappa}$ = probability that last interaction was at τ



Recombination

$$H^+ + e^- \longleftrightarrow H + \gamma$$

- accurate simulation extremely involved (many excited levels contribute)

Recombination

$$\mathrm{H}^+ + \mathrm{e}^- \longleftrightarrow \mathrm{H} + \gamma$$

- accurate simulation extremely involved (many excited levels contribute)
- RECFAST: Peebles recombination (two levels) with fudge factors

Recombination

$$\mathrm{H}^+ + \mathrm{e}^- \longleftrightarrow \mathrm{H} + \gamma$$

- accurate simulation extremely involved (many excited levels contribute)
- RECFAST: Peebles recombination (two levels) with fudge factors
- also HyRec, CosmoRec. All agree with RECAST v1.6 at precision level of Planck

Recombination

$$H^+ + e^- \longleftrightarrow H + \gamma$$

- accurate simulation extremely involved (many excited levels contribute)
- RECFAST: Peebles recombination (two levels) with fudge factors
- also HyRec, CosmoRec. All agree with RECAST v1.6 at precision level of Planck
- CLASS user can switch between RECFAST 1.6 (coded within `source/thermodynamics.c`) and HyRec (separate code in `hyrec/`) using `recombination = RECFAST` or `recombination = HyRec`.

Recombination

$$\mathrm{H}^+ + \mathrm{e}^- \longleftrightarrow \mathrm{H} + \gamma$$

- accurate simulation extremely involved (many excited levels contribute)
- RECFAST: Peebles recombination (two levels) with fudge factors
- also HyRec, CosmoRec. All agree with RECAST v1.6 at precision level of Planck
- CLASS user can switch between RECFAST 1.6 (coded within `source/thermodynamics.c`) and HyRec (separate code in `hyrec/`) using `recombination = RECFAST` or `recombination = HyRec`.
- RECFAST integrates $\frac{d}{dz}\{x_{\rm H}, x_{\rm He}, T_b\}$. HyRec is more sophisticated.

Recombination

$$\mathrm{H}^+ + \mathrm{e}^- \longleftrightarrow \mathrm{H} + \gamma$$

- accurate simulation extremely involved (many excited levels contribute)
- RECFAST: Peebles recombination (two levels) with fudge factors
- also HyRec, CosmoRec. All agree with RECAST v1.6 at precision level of Planck
- CLASS user can switch between RECFAST 1.6 (coded within `source/thermodynamics.c`) and HyRec (separate code in `hyrec/`) using `recombination = RECFAST` or `recombination = HyRec`.
- RECFAST integrates $\frac{d}{dz}\{x_{\rm H}, x_{\rm He}, T_b\}$. HyRec is more sophisticated.
- In both cases, CLASS needs to keep in memory an interpolation table for just $\{x_e(z), T_b(z)\}$.

Recombination

- Recombination needs one more cosmological parameter: the primordial Helium fraction $Y_{\rm He}$.
- User can fix it to given value (e.g. `Y_He = 0.25`) or to `Y_He = BBN`. Then the value is infered from an interpolation table computed with a BBN code (Parthenope), for each given value of $N_{\rm eff}$, ω_b (assumes $\mu_{\nu_e} = 0$, easy to generalise).
- BBN interpolation table located in separate directory, in `bbn/bbn.dat`

The module thermodynamics.c

External functions are:

- `thermodynamics_at_z(pba,pth,z,...,pvecthermo)`: interpolates in thermodynamics table (stored in `pth`) at a given z, returns a vector `pvecthermo`.
- `thermodynamics_init(ppr,pba,pth)`: computes thermodynamics table and stores it in `pth`.
- `thermodynamics_free(pth)`: free memory allocated in `pth`.

Let us now review the many tasks of `thermodynamics_init()`.

The function thermodynamics_init(ppr,pba,pth)

- define all indices with `thermodynamics_indices()`

The function thermodynamics_init(ppr,pba,pth)

- define all indices with `thermodynamics_indices()`
- eventually, find $Y_{\rm He}$ with `thermodynamics_helium_from_bbn()`

The function thermodynamics_init(ppr,pba,pth)

- define all indices with `thermodynamics_indices()`
- eventually, find $Y_{\rm He}$ with `thermodynamics_helium_from_bbn()`
- solve recombination with `thermodynamics_recombination()`, which calls either RECFAST or HyRec, and stores $\{x_e(z), T_b(z)\}$ (and derived quantities) in a temporary structure `preco`.

The function thermodynamics_init(ppr,pba,pth)

- define all indices with `thermodynamics_indices()`
- eventually, find $Y_{\rm He}$ with `thermodynamics_helium_from_bbn()`
- solve recombination with `thermodynamics_recombination()`, which calls either RECFAST or HyRec, and stores $\{x_e(z), T_b(z)\}$ (and derived quantities) in a temporary structure `preco`.
- impose a function $x_e(z)$ at small redshift, following user's input, but ensuring continuity with the solution from recombination. Store low-z $\{x_e(z), T_b(z)\}$ (and derived quantities) in a temporary structure `preio`.

The function thermodynamics_init(ppr,pba,pth)

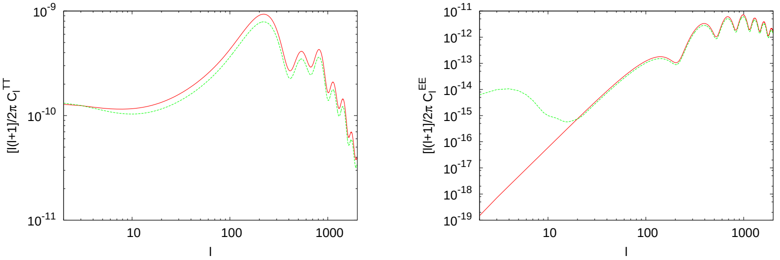
- define all indices with `thermodynamics_indices()`
- eventually, find $Y_{\rm He}$ with `thermodynamics_helium_from_bbn()`
- solve recombination with `thermodynamics_recombination()`, which calls either RECFAST or HyRec, and stores $\{x_e(z), T_b(z)\}$ (and derived quantities) in a temporary structure `preco`.
- impose a function $x_e(z)$ at small redshift, following user's input, but ensuring continuity with the solution from recombination. Store low-z $\{x_e(z), T_b(z)\}$ (and derived quantities) in a temporary structure `preio`.
- `thermodynamics_merge_reco_and_reio()` fills the final interpolation table in `pth` using `preco` at high z and `preio` at low z.

The function thermodynamics_init(ppr,pba,pth)

- define all indices with `thermodynamics_indices()`
- eventually, find $Y_{\rm He}$ with `thermodynamics_helium_from_bbn()`
- solve recombination with `thermodynamics_recombination()`, which calls either RECFAST or HyRec, and stores $\{x_e(z), T_b(z)\}$ (and derived quantities) in a temporary structure `preco`.
- impose a function $x_e(z)$ at small redshift, following user's input, but ensuring continuity with the solution from recombination. Store low-z $\{x_e(z), T_b(z)\}$ (and derived quantities) in a temporary structure `preio`.
- `thermodynamics_merge_reco_and_reio()` fills the final interpolation table in `pth` using `preco` at high z and `preio` at low z.
- derivation of a few more related quantities (see later).

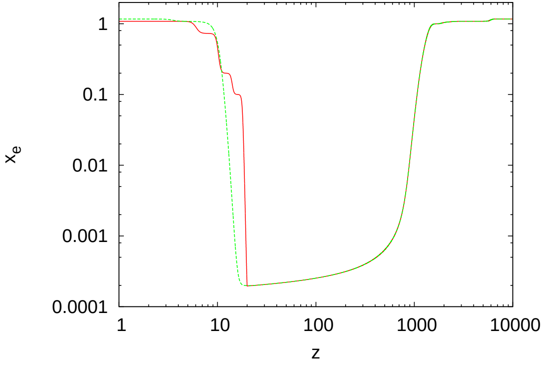
Reionization models

- reionisation very uncertain. Can be probed directly by looking at IGM (Lyman-α, ...) but with large uncertainties.
- CMB probes mainly an integrated quantity, $\tau_{\rm reio} = \int_{\tau_*}^{\tau_0} \kappa' d\tau$, close to 0.08. Gives suppression of C_l's at large l due to rescattering.
- small-l CMB (T and even better E) gives information on history (i.e. on $x_e(z)$, through $\kappa'(z)$).



Reionization models

- if `reio_parametrization = reio_camb`, $x_e(z)$ has a *$tanh$-shaped step*, centered on $z_{\rm reio}$, and matched to the correct value corresponding to freeze-out after recombination. User free to pass either `z_reio = ...` or `tau_reio = ...`. Codes find the missing one automatically, stores it in `pth` (and indicates it in output if `thermodynamics_verbose` > 0).

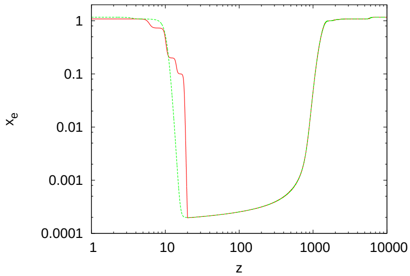


Reionization models

- instead, if `reio_parametrization = reio_bins_tanh`, code assumes a binned reionisation history, with smooth $tanh$ steps between bin centers. User passes e.g.

```
binned_reio_num = 3
binned_reio_z = 8,12,16
binned_reio_xe = 0.7,0.2,0.1
binned_reio_step_sharpness = 0.3
```

- then `tau_reio` cannot be passed in input, but calculated, stored and given in output.



Quantities stored in thermodynamics_table

The table `pth->thermodynamics_table[index_z*pth->th_size+pba->index_th]` has indices:

`index_th_xe`	ionization fraction	x_e
`index_th_dkappa`	Thomson scattering rate	κ' (units Mpc^{-1})
`index_th_tau_d`	Baryon drag optical depth	$\int_\tau^{\tau_0} \frac{4\rho_\gamma}{3\rho_b} \kappa' d\tau$
`index_th_exp_m_kappa`	exp. of (photon) optical depth	$e^{-\kappa}$ with $\kappa = \int_\tau^{\tau_0} \kappa' d\tau$
`index_th_g`	visibility function	$g = \kappa' e^{-\kappa}$
`index_th_Tb`	baryon temperature	T_b given by RECFAST
`index_th_cb2`	squared baryon sound speed	$c_b^2 = \frac{k_B}{\mu} T_b \left(1 - \frac{1}{3} \frac{d \ln T_b}{d \ln a}\right)$
`index_th_rate`	max. variation rate	(for sampling the sources)

(plus extra indices for other derivatives: κ'', κ''', g', g'', $(c_b^2)'$, $(c_b^2)''$).

Getting thermodynamical quantities from other modules

First allocate background and thermodynamics vectors:

```
double * pvecback;
double * pvecthermo;

class_alloc(pvecback,
            pba->bg_size_short*sizeof(double),
            ppt->error_message);

class_alloc(pvecthermo,
            pth->th_size*sizeof(double),
            ppt->error_message);
```

Getting thermodynamical quantities from other modules

Then, fill them:

```
class_call(background_at_tau(pba,
                             tau,
                             pba->short_info,
                             ..., ...,
                             pvecback),
                             pba->error_message,
                             ppt->error_message);

class_call(thermodynamics_at_z(pba,
                               pth,
                               z,
                               ..., ...,
                               pvecback,
                               pvecthermo),
                               pth->error_message,
                               ppt->error_message);

if (pvecthermo[pth->index_th_dkappa] > H) {...}
```

(pvecback needed to extrapolate accurately for $z > z_{\mathrm{recfast\ max.}}$)

Other useful quantities stored in pth

- `thermodynamics_init()` finds $z_{\rm rec}$ numerically, by searching for the maximum of the visibility function.

Other useful quantities stored in pth

- thermodynamics_init() finds z_{rec} numerically, by searching for the maximum of the visibility function.
- it stores the related quantities pth->z_rec, pth->tau_rec, pth->rs_rec, pth->ds_rec, pth->ra_rec, pth->da_rec.

Other useful quantities stored in pth

- `thermodynamics_init()` finds $z_{\rm rec}$ numerically, by searching for the maximum of the visibility function.
- it stores the related quantities `pth->z_rec`, `pth->tau_rec`, `pth->rs_rec`, `pth->ds_rec`, `pth->ra_rec`, `pth->da_rec`.
- These quantitites play a crucial role in choosing the sampling of the sources in k-space, because oscillation phase given by $\cos\left(2\pi \frac{d_s(z_{\rm rec})}{\lambda(z_{\rm rec})}\right)$. May give an estimate of $\theta_{\rm peak} = \frac{\pi}{l_{\rm peak}} \sim \theta_s \equiv \frac{d_s(z_{\rm rec})}{d_a(z_{\rm rec})} = \frac{r_s(z_{\rm rec})}{r_a(z_{\rm rec})}$.

Other useful quantities stored in pth

- `thermodynamics_init()` finds $z_{\rm rec}$ numerically, by searching for the maximum of the visibility function.
- it stores the related quantities `pth->z_rec`, `pth->tau_rec`, `pth->rs_rec`, `pth->ds_rec`, `pth->ra_rec`, `pth->da_rec`.
- These quantitites play a crucial role in choosing the sampling of the sources in k-space, because oscillation phase given by $\cos\left(2\pi \frac{d_s(z_{\rm rec})}{\lambda(z_{\rm rec})}\right)$. May give an estimate of $\theta_{\rm peak} = \frac{\pi}{l_{\rm peak}} \sim \theta_s \equiv \frac{d_s(z_{\rm rec})}{d_a(z_{\rm rec})} = \frac{r_s(z_{\rm rec})}{r_a(z_{\rm rec})}$.
- also finds the baryon drag time z_d numerically, such that the baryon optical depth at z_d is one.

Other useful quantities stored in pth

- `thermodynamics_init()` finds $z_{\rm rec}$ numerically, by searching for the maximum of the visibility function.
- it stores the related quantities `pth->z_rec`, `pth->tau_rec`, `pth->rs_rec`, `pth->ds_rec`, `pth->ra_rec`, `pth->da_rec`.
- These quantitites play a crucial role in choosing the sampling of the sources in k-space, because oscillation phase given by $\cos\left(2\pi \frac{d_s(z_{\rm rec})}{\lambda(z_{\rm rec})}\right)$. May give an estimate of $\theta_{\rm peak} = \frac{\pi}{l_{\rm peak}} \sim \theta_s \equiv \frac{d_s(z_{\rm rec})}{d_a(z_{\rm rec})} = \frac{r_s(z_{\rm rec})}{r_a(z_{\rm rec})}$.
- also finds the baryon drag time z_d numerically, such that the baryon optical depth at z_d is one.
- it stores the related quantities `pth->z_d`, `pth->tau_d`, `pth->ds_d`, `pth->rs_d` (the latter gives the phase of the BAOs in large scale structure).

Is RECFAST identical in CLASS and CAMB?

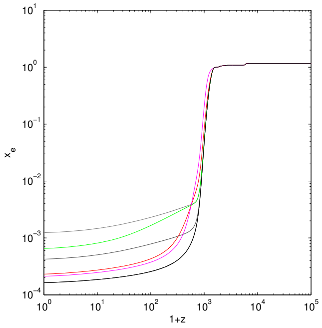
Two differences:

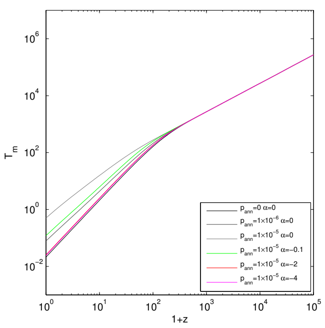
- RECFAST solution slightly smoothed around points where solution is not derivable. Just useful for testing the limit of high accuracy / small stepsize in RECFAST.

Is RECFAST identical in CLASS and CAMB?

Two differences:

- RECFAST solution slightly smoothed around points where solution is not derivable. Just useful for testing the limit of high accuracy / small stepsize in RECFAST.
- several input parameters allow to play with a DM annihilation effect, as described in Giesen et al. 2012. Effect on x_e and T_b, with signatures on CMB.





Printing the thermal history

Execute e.g. `./class myinput.ini` including in the input file:

```
write thermodynamics = yes
root = output/toto_
```

Printing the thermal history

Execute e.g. `./class myinput.ini` including in the input file:

```
write thermodynamics = yes
root = output/toto_
```

The output module will also write a file `output/toto_thermodynamics.dat` with an explicit header:

```
# Table of selected thermodynamics quantitites
# The following notation is used in column titles:
# x_e = electron ionisation fraction
# -kappa = optical depth
# kappa' = Thomson scattering rate, prime denotes conformal time
derivatives
# g = kappa' e^-kappa = visibility function
# Tb = baryon temperature
# c_b^2 = baryon sound speed squared
# tau_d = baryon drag optical depth
#z conf. time [Mpc] x_e kappa' [Mpc^-1] exp(-kappa)g [Mpc^-1] Tb [K] c_b
^2 tau_d
```

Printing the thermal history

Execute e.g. `./class myinput.ini` including in the input file:

```
write thermodynamics = yes
root = output/toto_
```

The output module will also write a file `output/toto_thermodynamics.dat` with an explicit header:

```
# Table of selected thermodynamics quantitites
# The following notation is used in column titles:
# x_e = electron ionisation fraction
# -kappa = optical depth
# kappa' = Thomson scattering rate, prime denotes conformal time
derivatives
# g = kappa' e^-kappa = visibility function
# Tb = baryon temperature
# c_b^2 = baryon sound speed squared
# tau_d = baryon drag optical depth
#z conf. time [Mpc] x_e kappa' [Mpc^-1] exp(-kappa)g [Mpc^-1] Tb [K] c_b
^2 tau_d
```

Output easy to customise in `output.c`, by editing:
`output_one_line_of_thermodynamics(...)` for the quantities to plot in each line
`output_open_thermodynamics_file(...)` for the header (description of columns)
(another way would be to use `test_thermodynamics.c`)