Focus on: Non-Cold Dark Matter

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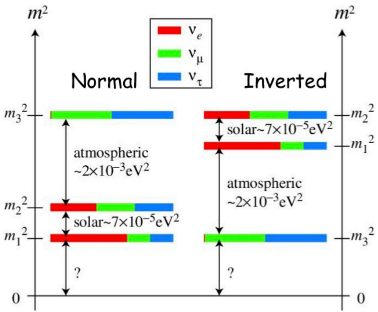
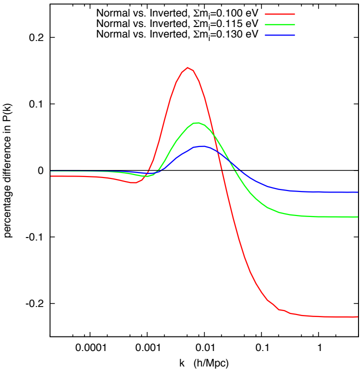
Non-Cold Dark Matter

- Non-Cold: species with a velocity dispersion that cannot be neglected, justifying Boltzmann equation rather than fluid equations
- Dark Matter: the species is not interacting at the epoch relevant for CMB and LSS. Coupled dark matter must be treated separately (see implementation in CLASS of DM-γ interaction in Wilkinson et al. `arXiv:1309.7588`, and DM-ν interaction in Wilkinson et al. `arXiv:1401.7597`).
- we leave the phase-space distribution (psd) arbitrary. Applications: ordinary massive neutrinos, massive neutrinos with non-standard production mechanism, warm dark matter, sterile neutrinos, etc.

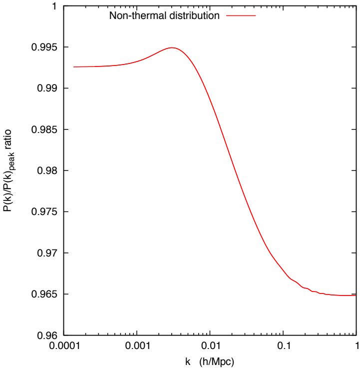
Impact of Normal versus inverted hierarchy on $P(k)$:





Impact of non-thermal distortion (bump produced by late decay):

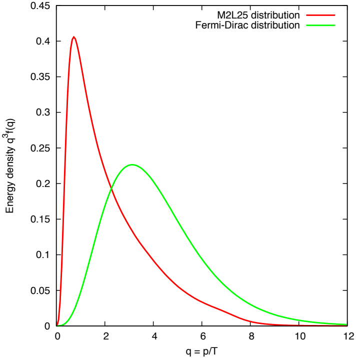
$$f(q) = \frac{2}{(2\pi)^3}\left[\frac{1}{e^q+1} + \frac{A\pi^2}{q^2\sqrt{2\pi}\sigma}\exp\left(-\frac{(q-q_c)^2}{2\sigma^2}\right)\right]$$

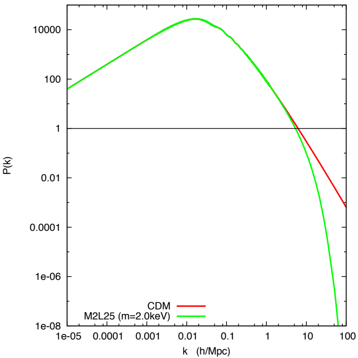


for 3 degenerate neutrinos with $m = 1$ eV, $A = 0.018$, $\sigma = 1$, $q_c = 10.5$, compared with 3 degenerate thermal massive neutrinos with same mass and $N_{\rm eff}$.

NCDM Gallery

Impact of non-thermal distortion (sterile neutrino WDM produced resonantly, psd passed in file):





for sterile neutrino WDM with $m = 2$ keV, compared with CDM with the same ω_{DM}.

The Boltzmann equation

The fundamental equation in cosmology:

$$\mathcal{L}\left[f_i(\tau, \vec{x}, \vec{p})\right] = \mathcal{C}\left[f_i, f_j\right] (= 0)$$

- First order perturbation theory: $f_i = f_0(1+\Psi)$
- Boltzmann equation for Ψ:

$$\frac{\partial \Psi}{\partial \tau} + i\frac{q}{\epsilon}(\vec{k}\cdot\hat{n}) + \frac{d\ln f_0}{d\ln q}\left[\phi' - i\frac{\epsilon}{q}(\vec{k}\cdot\hat{n})\psi\right] = 0 \ ,$$

$$\vec{q} = \vec{p}/T_{\rm ncdm} \ , \qquad \epsilon = \sqrt{p^2+m^2}/T_{\rm ncdm}$$

  (Ma & Bertschinger 92). Here $T_{\rm ncdm}$ is a normalising quantity scaling like a^{-1}, but the species does not need to be thermalised.
- $\vec{k}\cdot\hat{n} = \cos\theta$, no dependence on φ, Legendre expansion!

The Boltzmann equation

- We expand Ψ in Legendre multipoles:

$$\Psi = \sum_{l=0}^{\infty} (-i)^l (2l+1) \Psi_l P_l(\cos\theta) \ .$$

- Boltzmann hierarchy:

$$\Psi_0' = -\frac{qk}{\epsilon}\Psi_1 - \phi' \frac{d\ln f_0}{d\ln q} \ ,$$

$$\Psi_1' = \frac{qk}{3\epsilon}(\Psi_0 - 2\Psi_2) - \frac{\epsilon k}{3q}\psi \frac{d\ln f_0}{d\ln q} \ ,$$

$$\Psi_l' = \frac{qk}{(2l+1)\epsilon}\left[l\Psi_{l-1} - (l+1)\Psi_{l+1}\right] \ , l \geq 1 \ .$$

$T^{\mu\nu}$ contribution

NCDM coupled to other species **only** through integrated quantitates:

$$\delta\rho_{\rm ncdm} = 4\pi T^4_{\rm ncdm} \int q^2 dq\, f_0 \epsilon \Psi_0$$

$$\delta p_{\rm ncdm} = \frac{4\pi}{3} T^4_{\rm ncdm} \int q^2 dq\, f_0 \frac{q^2}{\epsilon} \Psi_0$$

$$(\bar\rho + \bar p)\theta_{\rm ncdm} = 4\pi k T^4_{\rm ncdm} \int q^2 dq\, f_0 q \Psi_1$$

$$(\bar\rho + \bar p)\sigma_{\rm ncdm} = \frac{8\pi}{3} T^4_{\rm ncdm} \int q^2 dq\, f_0 \frac{q^2}{\epsilon} \Psi_2$$

How de we choose the vector q_i?

Adaptative quadrature scheme: the lecture on numerical methods will show how to sample q_i in an optimal way, adapting automatically to any new phase-space distribution, for given (dimensionless) precision parameter `tol_ncdm`. For Fermi-Dirac, accurate results with 5 points only! (much more accurate than old linear sampling with 14 points).

Fluid approximations

- can devise schemes for cutting hierarchy at some low l (keeping two or three equations)
- equivalent to imperfect fluid approximation with some recipes giving δp, σ as a function of the background quantitates and $\delta\rho$, θ
- different fluid approximation for massive neutrinos in Hu `astro-ph/9801234`; Shoji & Komatsu `arXiv:1003.0942`; Lesgourgues & Tram `arXiv:1104.2933` (CLASS IV paper).
- none of them accurate enough for being substituted to Boltzmann at all times!
- CLASS makes the substitution for each k, when $k\tau$ is below threshold (sub-Hubble regime)
- `ufa` for ultra-relativistic species, `ncdmfa` for the rest
- user can switch between different schemes; default $=$ CLASS II & IV paper approach; can change the threshold value.

Input parameters

Distribution can be read in file or analytic,

$$f_0(q) = \frac{\chi}{2\pi^3}\left[\frac{1}{e^{q-\xi}+1} + \frac{1}{e^{q+\xi}+1}\right]$$

```
N_ncdm = 3
# list of 0 and 1, 0 = analytic, 1 = from file, either one or N_ncdm
entries:
use_ncdm_psd_files = 0
# either one or the number of "from file" species:
ncdm_psd_filenames = psd_FD_single.dat
# either one or the number of "analytic" species:
m_ncdm = 0.04, 0.01, 0.001
Omega_ncdm =
omega_ncdm =
T_ncdm = #ratio T_ncdm/T_gamma
ksi_ncdm =
deg_ncdm =
```

- If you pass one of `m_ncdm`, `Omega_ncdm`, `omega_ncdm`, mass inferred from density or density from mass, assuming read value of `deg_ncdm` (set to one by default).
- If you pass one of `m_ncdm` AND one of `Omega_ncdm`, `omega_ncdm`, `deg_ncdm` is found automatically by the code.

Input parameters

Default value of T_ncdm is $(4/11)^{1/3}$ of instantaneous neutrino decoupling. For high precision: no value is exact, true neutrino psd slightly non-thermal. Hence no value of T_ncdm can account exactly for $N_{\rm eff} = 3.046$ and $\omega_\nu = \sum m_\nu/93.14$ eV. However, recommended value T_ncdm = 0.715985 achieves second relation, and is sufficient for fitting CMB/LSS.

In background_verbose mode ≥ 1 (run with N_ncdm = 1 , T_ncdm = 0.715985, m_ncdm = 0.06, deg_ncdm = 3):

```
-> ncdm species i=1 sampled with 11 (resp. 5)points for purpose of
background (resp. perturbation)integration. In the relativistic limit it
 gives N_eff = 3.03747
-> non-cold dark matter species with i=1 has m_i = 6.000000e-02 eV (so
m_i / omega_i =3.104537e+01 eV)
```

Input parameters

One can pass optional parameters for the analytic distribution function. See description of 'ncdm_psd_parameters' in `explanatory.ini`, and corresponding lines in `background.c`.

Example implemented in the code (commented region of `background.c`): take FD distribution in flavour space, projected to mass eigenstates, for given values of the mixing angles...