NCDM - Non-Cold Dark Matter

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NCDM explained

What we mean by NCDM

- **Non-Cold**: At sufficiently early times, the species can be assumed massless
- **Dark**: The particle is non-interacting for the epoch relevant for the CMB.
- **Note**: We are not assuming anything about the distribution function for the species.
- **Examples**: Massive Neutrinos, Warm Dark Matter, Sterile Neutrinos, ...

Not a fluid!

NCDM species are **not** a fluid! We must solve the Boltzmann equation.

The Boltzmann equation

The second most important equation in cosmology

- At an abstract level we can write:

$$\mathcal{L}\left[f_i(\tau, \mathbf{x}, \mathbf{p})\right] = \mathcal{C}\left[f_i, f_j\right](= 0). \qquad (1)$$

  The last equal sign is true for a **collisionless** species.

- We expand f_i to first order:

$$f_i(\tau, \mathbf{x}, \mathbf{p}) \simeq f_0(q)(1 + \Psi(\tau, \mathbf{x}, q, \hat{n})). \qquad (2)$$

- Plugging equation (2) into equation (1) gives a Boltzmann equation for Ψ in Fourier space:

$$\frac{\partial \Psi}{\partial \tau} + i\frac{qk}{\epsilon}(\mathbf{k} \cdot \hat{n})\Psi + \frac{d \ln f_0}{d \ln q}\left[\dot{\eta} - \frac{\dot{h} + 6\dot{\eta}}{2}(\hat{k} \cdot \hat{n})^2\right] = 0$$

Legendre expansion

A few missing definitions

We have defined the **comoving momentum** q and **comoving energy** ϵ by $q \equiv \frac{p}{T_{\rm ncdm}}$ and $\epsilon \equiv \frac{\sqrt{p^2+m^2}}{T_{\rm ncdm}}$.

The expansion of Ψ

Since $\hat{k} \cdot \hat{n} = \cos\theta$, the equation from before

$$\frac{\partial \Psi}{\partial \tau} + i\frac{qk}{\epsilon}(\mathbf{k} \cdot \hat{n})\Psi + \frac{d\ln f_0}{d\ln q}\left[\dot{\eta} - \frac{\dot{h}+6\dot{\eta}}{2}(\hat{k} \cdot \hat{n})^2\right] = 0$$

has no dependence on ϕ. Thus we can expand the angular dependence of Ψ in Legendre multipoles:

$$\Psi(\tau, k, q, \hat{k} \cdot \hat{n}) = \sum_{l}^{\infty}(2\ell+1)\Psi_l(\tau, k, q)P_\ell(\hat{k} \cdot \hat{n}) \qquad (3)$$

The Boltzmann hierarchy

$$\dot{\Psi}_0 = -\frac{qk}{\epsilon} + \frac{1}{6}\dot{h}\frac{d\ln f_0}{d\ln q},$$

$$\dot{\Psi}_1 = \frac{qk}{3\epsilon}(\Psi_0 - 2\Psi_2),$$

$$\dot{\Psi}_2 = \frac{qk}{5\epsilon}(2\Psi_1 - 3\Psi_3) - \left(\frac{1}{15}\dot{h} + \frac{2}{5}\dot{\eta}\right)\frac{d\ln f_0}{d\ln q},$$

$$\dot{\Psi}_\ell = \frac{qk}{(2\ell+1)\epsilon}\left(\ell\Psi_{\ell-1} - (\ell+1)\Psi_{\ell+1}\right), \quad \ell \leq 3.$$

The NCDM implementation

What are the features of our NCDM implementation?

- Any number of species, specified by `N_ncdm`.
- The distribution functions can be specified by
  1. specifying `deg_ncdm`, `T_ncdm`, `m_ncdm` and `ksi_ncdm`.
  2. a file containing a tabulated distribution function, filenames passed in `ncdm_psd_filenames`.
  3. Any analytic function containing any number of parameters passed through `ncdm_psd_parameters`.
- The q-sampling is automatic and depends on the actual distribution function.

Background distribution function

The default $f_0(q)$ is found in `background_ncdm_distribution()`

$$f_0(q) = \frac{1}{(2\pi)^3}\left[\frac{1}{\mathrm{e}^{q-\chi}+1} + \frac{1}{\mathrm{e}^{q+\chi}+1}\right]. \tag{4}$$

Customising the distribution function

background_ncdm_distribution()

```
/** -> deal first with the case of interpolating in files */
if (pba->got_files[n_ncdm]==_TRUE_) {

/** -> deal now with case of reading analytical function */
else{

  *f0 = 1./pow(2*_PI_,3)*(1./(exp(q-ksi)+1.)+1./(exp(q+ksi)+1.));

  if (_FALSE_) {

    /* extract values from the list (in this example, mixing
        angles) */
    double square_s12=param[0];
    double square_s23=param[1];
    double square_s13=param[2];

    for(i=0;i<3;i++)
      *f0 +=mixing_matrix[i][n_ncdm]*1./pow(2*_PI_,3)*(1./(exp(q-
        pba->ksi_ncdm[i])+1.)+1./(exp(q+pba->ksi_ncdm[i])+1.));
```

NCDM options

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```
6) all parameters describing the ncdm sector (i.e. any non-cold
    dark matter relics, including massive neutrinos, warm dark
    matter, etc.):

  -> 'N_ncdm' is the number of distinct species (default: set to 0)

N_ncdm = 0

  -> 'use_ncdm_psd_files' is the list of N_ncdm numbers: 0 means '
      phase-space distribution (psd) passed analytically inside the
       code, in the mnodule background.c, inside the function
      background_ncdm_distribution()'; 1 means 'psd passed as a
      file with at list two columns: first for q, second for f_0(q)
      ', where q is p/T_ncdm (default: only zeros)

#use_ncdm_psd_files = 0
```

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```
  -> if some of the previous values are equal to one,
      'ncdm_psd_filenames' is the list of names of
     psd files (as many as number of ones in
     previous entry)

ncdm_psd_filenames = psd_FD_single.dat

  -> 'ncdm_psd_parameters' is an optional list of
     double parameters to describe the analytic
     distribution function or to modify a p.s.d.
     passed as a file. It is made available in the
     routine background_ncdm_distribution.

#ncdm_psd_parameters = Nactive, sin^2_12 ,s23 ,s13
ncdm_psd_parameters = 0.3 ,0.5, 0.05
```

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```
The remaining parameters should be entered as a list
    of 'N_ncdm' numbers separated by commas:

  -> 'Omega_ncdm' or 'omega_ncdm' or 'm_ncdm' in eV (
      default: all set to zero) with only one of
      these inputs, CLASS computes the correct value
      of the mass; if both (Omega_ncdm, m_ncdm) or (
      omega_ncdm, m_ncdm) are passed, CLASS will
      renormalise the psd in order to fulfill both
      conditions.

     Passing zero in the list of m_ncdm's or
         Omeg_ncdm's means that for this component,
         this coefficient is not imposed, and its
         value is inferred from the other one.

m_ncdm = 0.04, 0.04, 0.04
Omega_ncdm =
```

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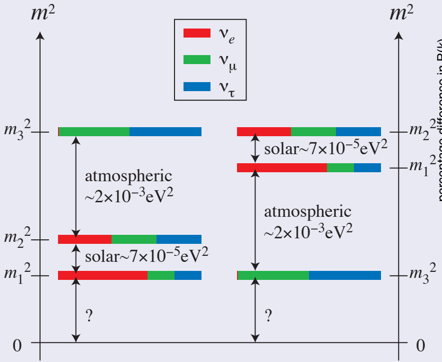
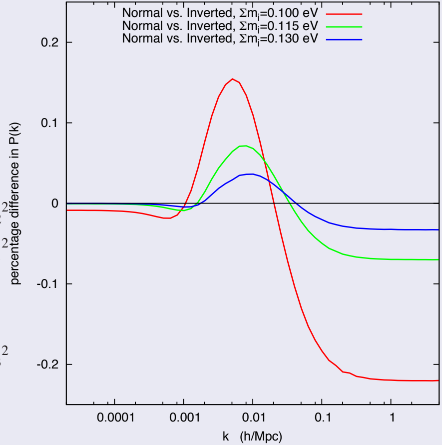
```
  -> 'T_ncdm' is the ncdm temperature in units of photon
     temperature (default:set to (4/11)^(1/3). Note that
     active massive neutrinos, the recommended default
     value is 0.71611 (this value is fudged to account for
     realistic neutrino decoupling: gives m/omega equal to
     93.14 eV, as in hep-ph/0506164)

#T_ncdm = 0.71611

  -> 'ksi_ncdm' is the ncdm chemical potential in units of
     its own temperature (default: set to 0)

ksi_ncdm =

  -> 'deg_ncdm' is the degeneracy parameter multiplying the
     psd: 1 stands for 'one family', i.e. one particle +
     anti-particle (default: set to 1.0)

deg_ncdm =
```

Exercise 1: Neutrino hierarchies

Normal versus inverted hierarchy





Exercise 1: Some hints

Getting the masses right

Assuming zero mass splitting between 2 states, we have:

$$\Sigma m = 2m_- + m_+, \quad m_+ = \sqrt{m_-^2 + |\delta^2_{\mathrm{atm.}}|} \qquad \text{(NH)},$$

$$\Sigma m = m_- + 2m_+, \quad m_+ = \sqrt{m_-^2 + |\delta^2_{\mathrm{atm.}}|} \qquad \text{(IH)}.$$

NH.ini

This .ini-file corresponds to $\sum m_i = 0.1\mathrm{eV}$ for Normal Hierarchy, NH:

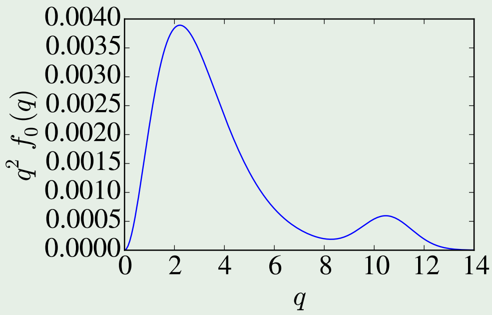
```
N_ur = 0.
N_ncdm = 2
m_ncdm = 0.02450296, 0.05099407
```

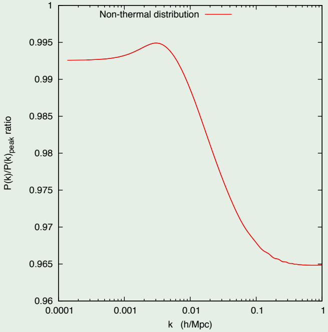
NH.ini cont.

```
deg_ncdm = 2, 1
P_k_max_1/Mpc = 3.
output = mPk
#ncdmfa = 3
```

Exercise 2: A peak in the distribution

The effect of a peak





Exercise 2: Hints

Fermi-Dirac with a Gaussian peak

$$f(q) = \frac{2}{(2\pi)^3}\left[\frac{1}{e^q+1} + \frac{A\pi^2}{q^2\sigma\sqrt{2\pi}}\exp\left(-\frac{(q-q_c)^2}{2\sigma^2}\right)\right] \quad (5)$$

decaypeak_ON.ini

May not be complete, but something like:

```
N_ncdm = 1
m_ncdm = 1.0
deg_ncdm = 3.0
ncdm_psd_parameters = 0.018, 1.0, 10.5
P_k_max_1/Mpc = 1.
output = mPk
```