Bayesian Parameter Extraction

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Outline

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Bibliography

Book

- Bayesian methods in cosmology (Hobson, Jaffe, Liddle, Mukherjee and Parkinson)

Review Articles

- Comparison of sampling techniques for Bayesian parameter estimation (*Rupert Allison, Joanna Dunkley*, arXiv:1308.2675)
- Bayes in the sky: Bayesian inference and model selection in cosmology (*Roberto Trotta*, arXiv:0803.4089)

Precision Cosmology

Precise measurement of random realisations

- Beginning of **precision** experiments with **WMAP**

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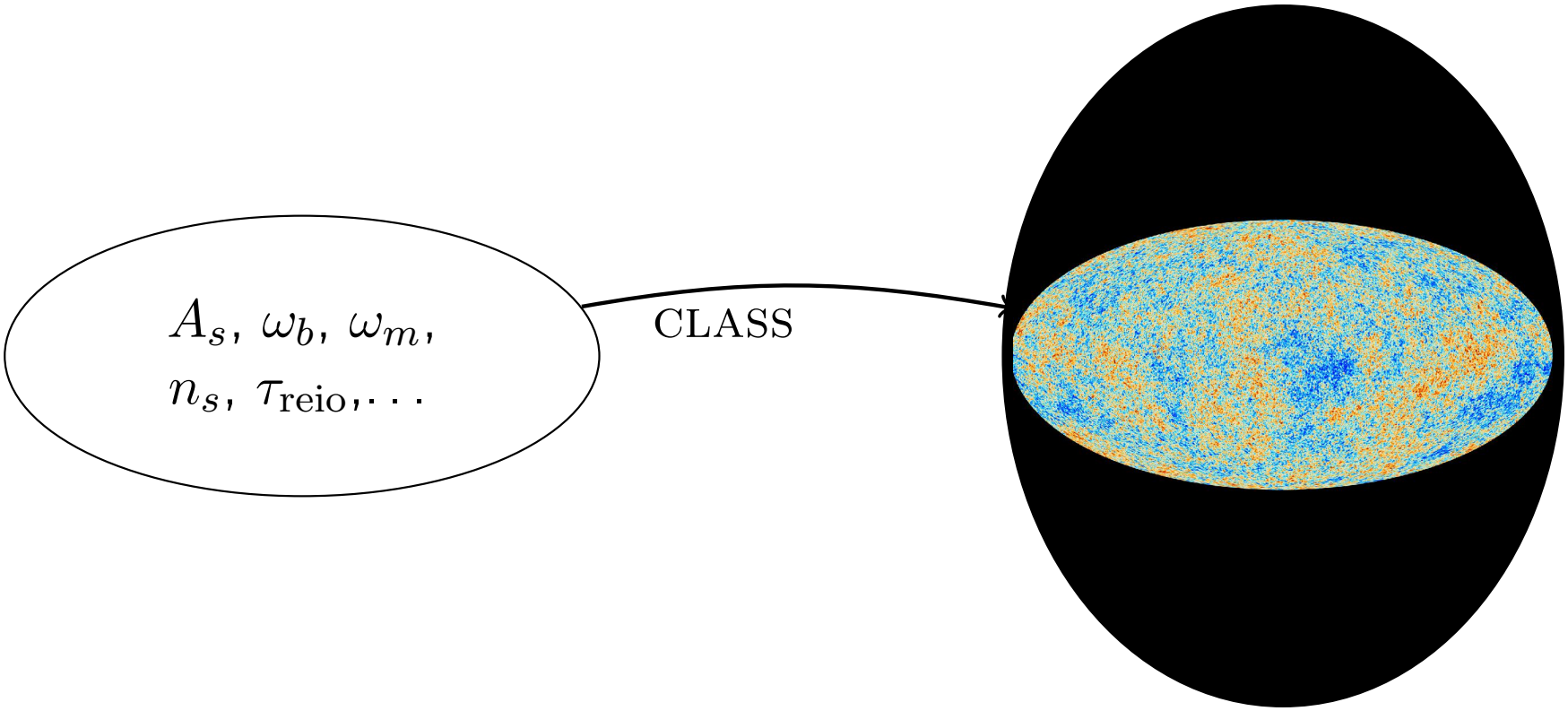
- How to **infer the value of the parameters from the data** ?

Similar to Particle Physics

Prediction only for, *e.g.* the rate of decay of a particle. Information acquired when statistically observing this decay channel (how many times did it decay to this particular product ?)

The big picture



Bayesian approach

from Bayesian Methods in Cosmology

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Bayesian methods

- All quantities are considered **statistical**
- The question we can answer is: **is this model better than another ?**
- We infer **credible regions** in which, given a model, the parameters live.
- Our knowledge depends on the **data measured**.

Bayesian approach

Definitions

Given a **context** $\mathcal{I}$ and a set of **data** D:

- θ: **continuous** values for a parameter set
- $\mathrm{pr}(\theta) \geq 0$: **probability** function is positive
- $\int \mathrm{pr}(\theta)\mathrm{d}\theta = 1$: **Sum rule**
- $\mathrm{pr}(\phi, \theta) = \mathrm{pr}(\phi|\theta)\mathrm{pr}(\theta)$: **Product rule**

$\Big\} \; || \; \mathcal{I}$

Bayes Theorem

Bayes Theorem

All quantities are given in the context $\mathcal{I}$

$\mathrm{pr}(\theta)\mathrm{pr}(D\|\theta)$	$=$	$\mathrm{pr}(\theta, D)$	$=$	$\mathrm{pr}(D)\mathrm{pr}(\theta\|D)$
Prior $\times$ **Likelihood**	$=$	**Joint**	$=$	**Evidence** $\times$ **Posterior**
$\pi(\theta)\mathcal{L}(\theta)$	$=$	$\ldots$	$=$	$E\mathcal{P}(\theta)$
Input		$\longrightarrow$		Output

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$$\begin{array}{ccccc} \mathrm{pr}(\theta)\mathrm{pr}(D|\theta) & = & \mathrm{pr}(\theta, D) & = & \mathrm{pr}(D)\mathrm{pr}(\theta|D) \\ \textbf{Prior} \times \text{ Likelihood} & = & \text{Joint} & = & \text{Evidence} \times \text{Posterior} \\ \pi(\theta)\mathcal{L}(\theta) & = & \dots & = & E\mathcal{P}(\theta) \\ \text{Input} & & \longrightarrow & & \text{Output} \end{array}$$

Prior

A priori information on the parameters. Most of the time, it is **flat** (uniform chance to be inside a given volume).

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Likelihood

Given by the instrument operated at *known input*. If uncontrolled unknow: **nuisance parameters**.

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Evidence

Only recovered with Nested Sampling, gives an information on how well the given context suits the data.

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Posterior Distribution

The name of the game. Inferred distribution of probability, after using the data.

Bayes Theorem

Complete Rules

$$\int \pi(\theta)\mathrm{d}\theta = \int \mathcal{P}(\theta)\mathrm{d}\theta = 1$$

$$E = \int \mathcal{L}(\theta)\pi(\theta)\mathrm{d}\theta$$

$$\mathcal{P}(\theta) = \frac{\pi(\theta)\mathcal{L}(\theta)}{E}$$

Bayesian approach: issues

What are the problems ?

- **Evidence** is hard to compute because...
- we don't usually know the **Likelihood** function (analytically).
- so we don't know where to **sample** it (many dimensions)...
- and it might be **computationally expensive**.

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How to deal with it ?

- we have to sample **randomly** the volume (*all methods*)
- we can avoid computing this integral by **computing ratios** (*mcmc*)

Methods that give the Evidence

Comparison between models

Evidence is **how well your context explains the data**. Nested Sampling gives this. Others dont. . . but best-fit likelihood gives some indication.

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Caution

These methods will only give you a **probable** answer, not a definite one. It is the price you pay for not doing your integral exactly.

But...

Living without the evidence

Theoretically motivated model: we want to know the values of this parameter to explain the data. Then, the **posterior** is an interesting quantity, and the **evidence** can be left aside temporarily.

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Nonetheless

Beware of the best-fit likelihood value !

Outline

Algorithm and Sampler

Foreword

Algorithm

Tells you how to **choose** which point you move, and if you **accept** a new point or not

Sampler

Tells you how to **move**, **select a** new point.

Sometimes used **interchangeably**

Algorithm and Sampler

Foreword

available in Monte Python

As of v2.0.0, you can use **MultiNest** (`nested sampling`, by Farhan Feroz & Mike Hobson), and the **CosmoHammer** (`emcee`, by Joel Akeret & Sebastian Seehars)

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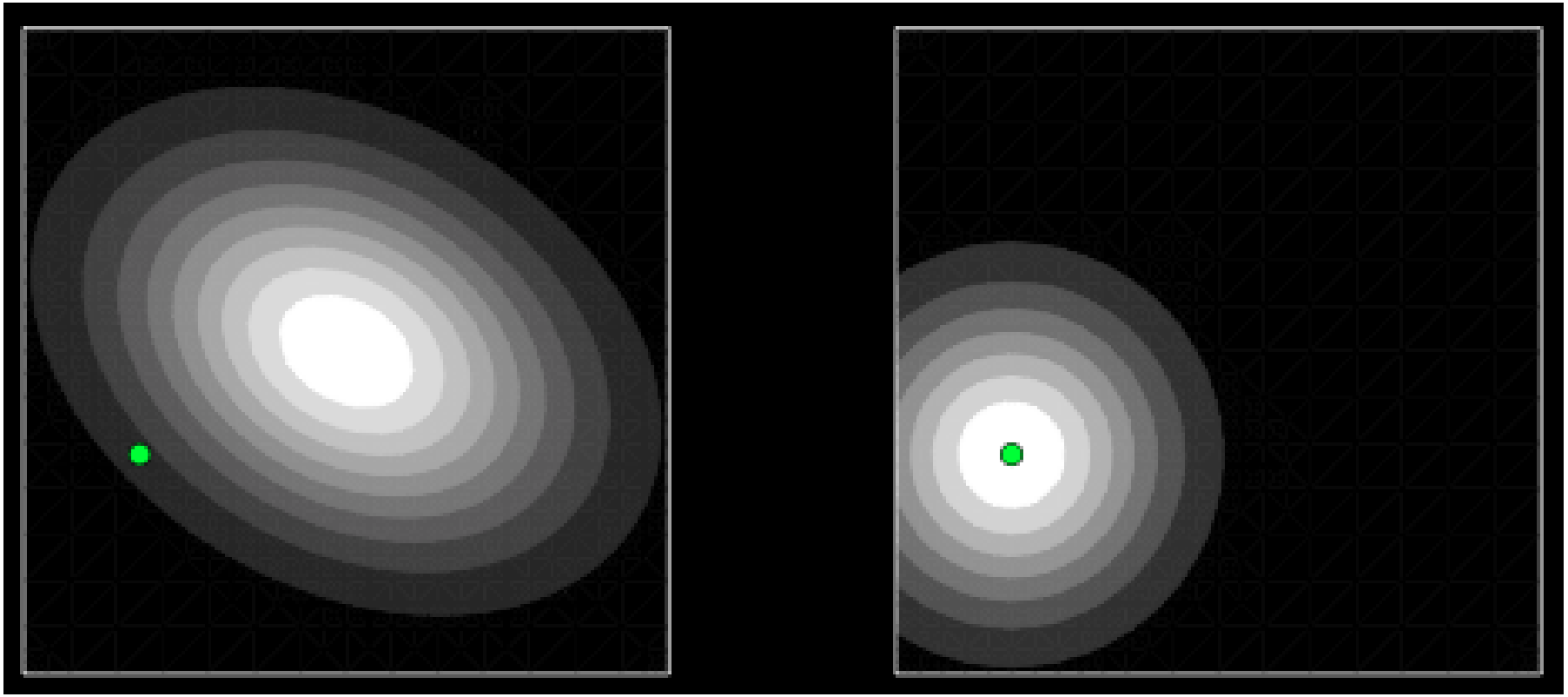
Cosmo Hammer

Thanks to Joel and Sebastian for helping setting this up

Metropolis-Hastings

Courtesy of Sebastian Seehars



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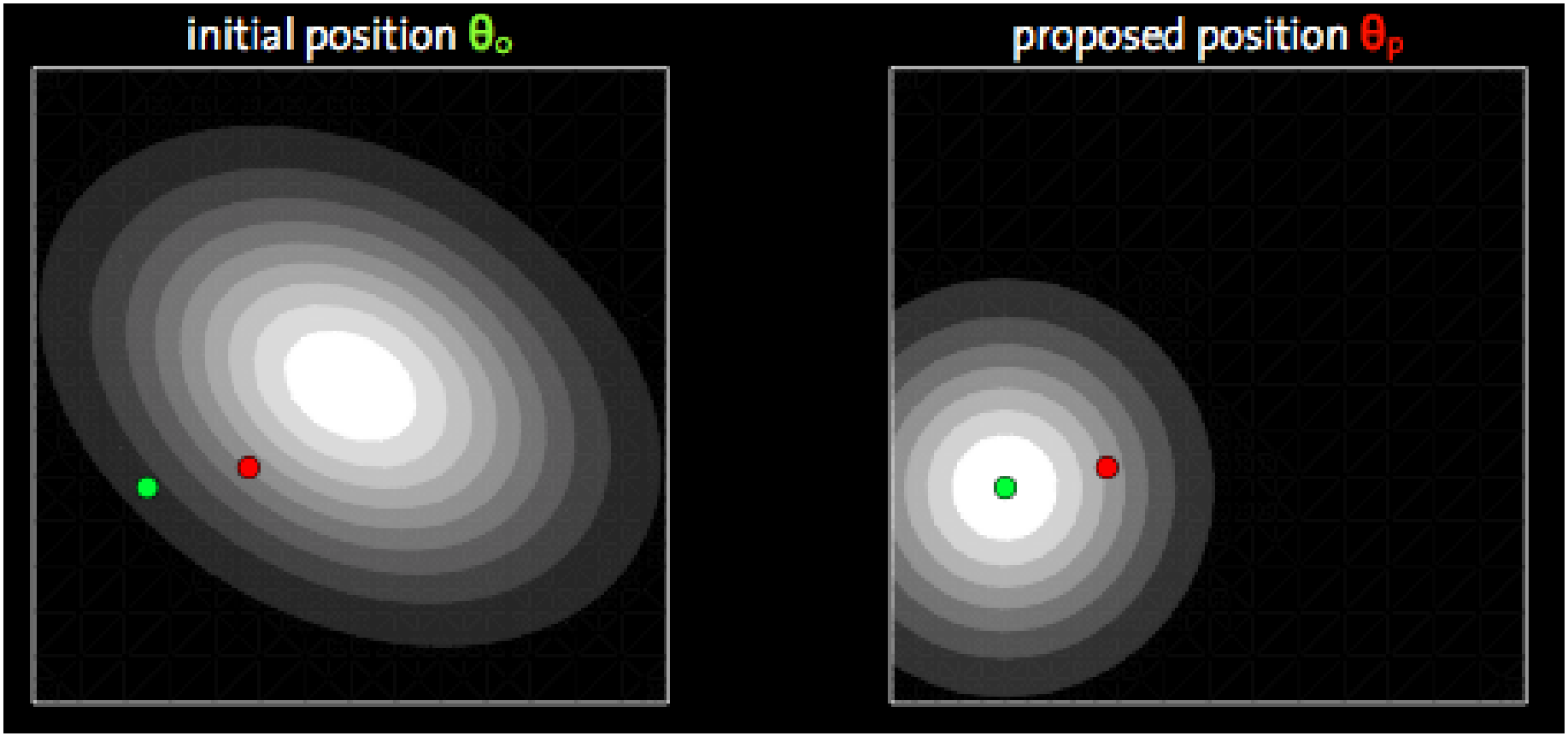

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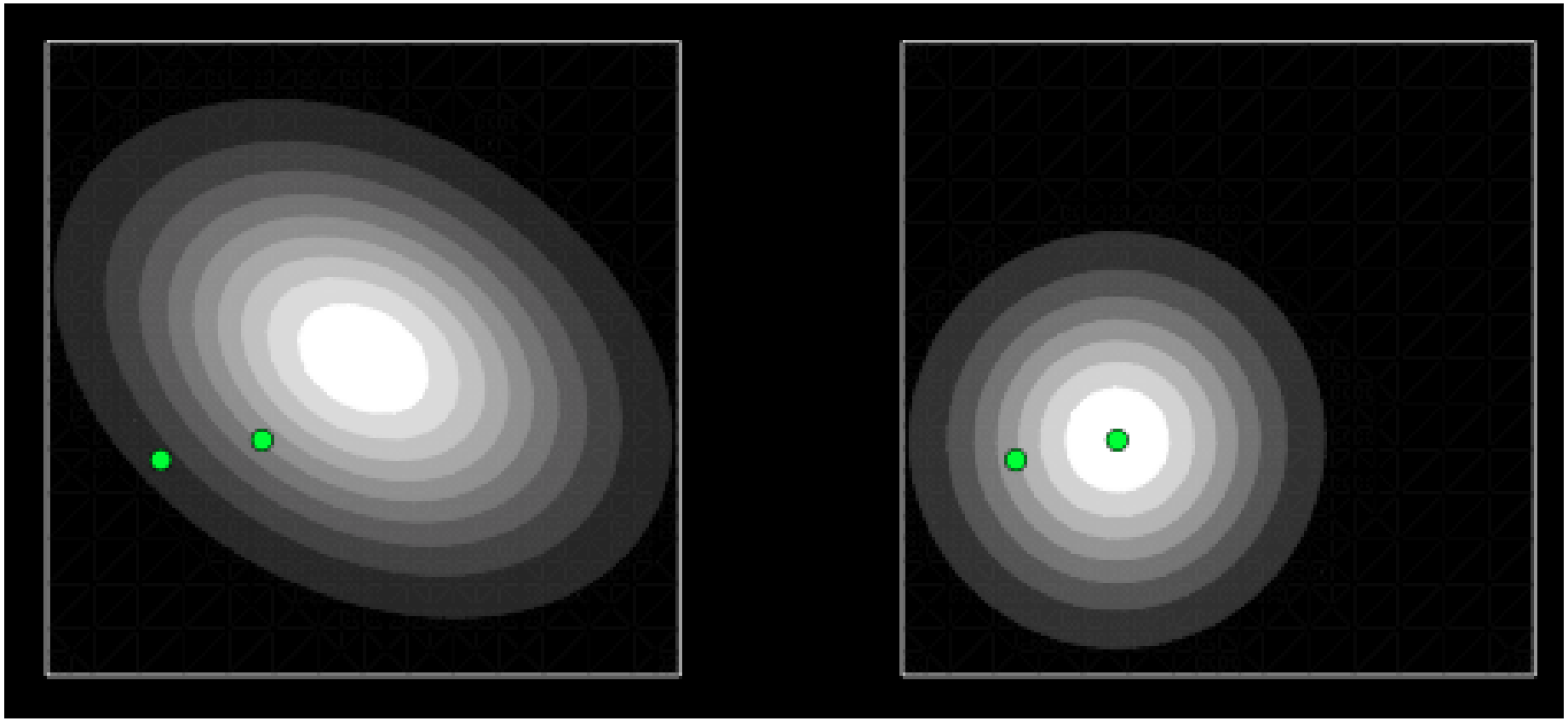

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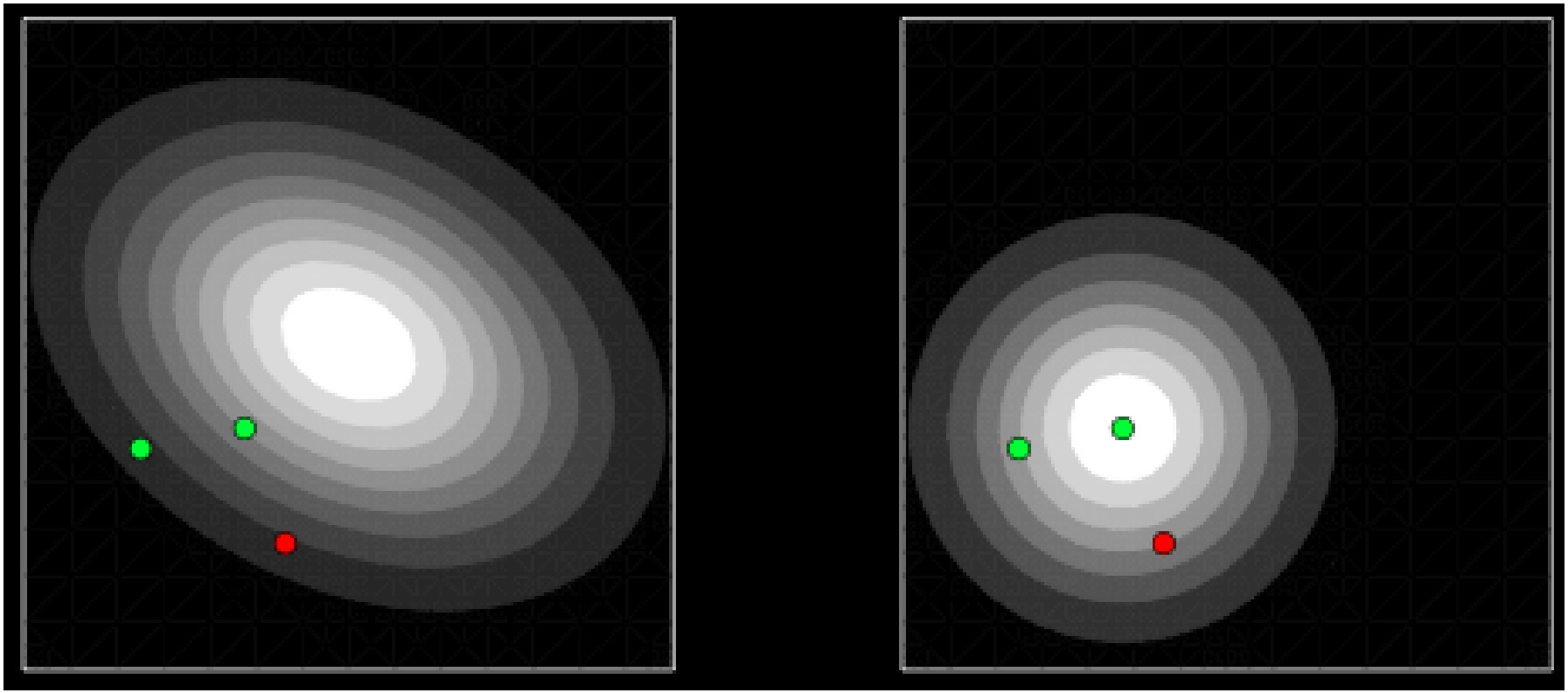

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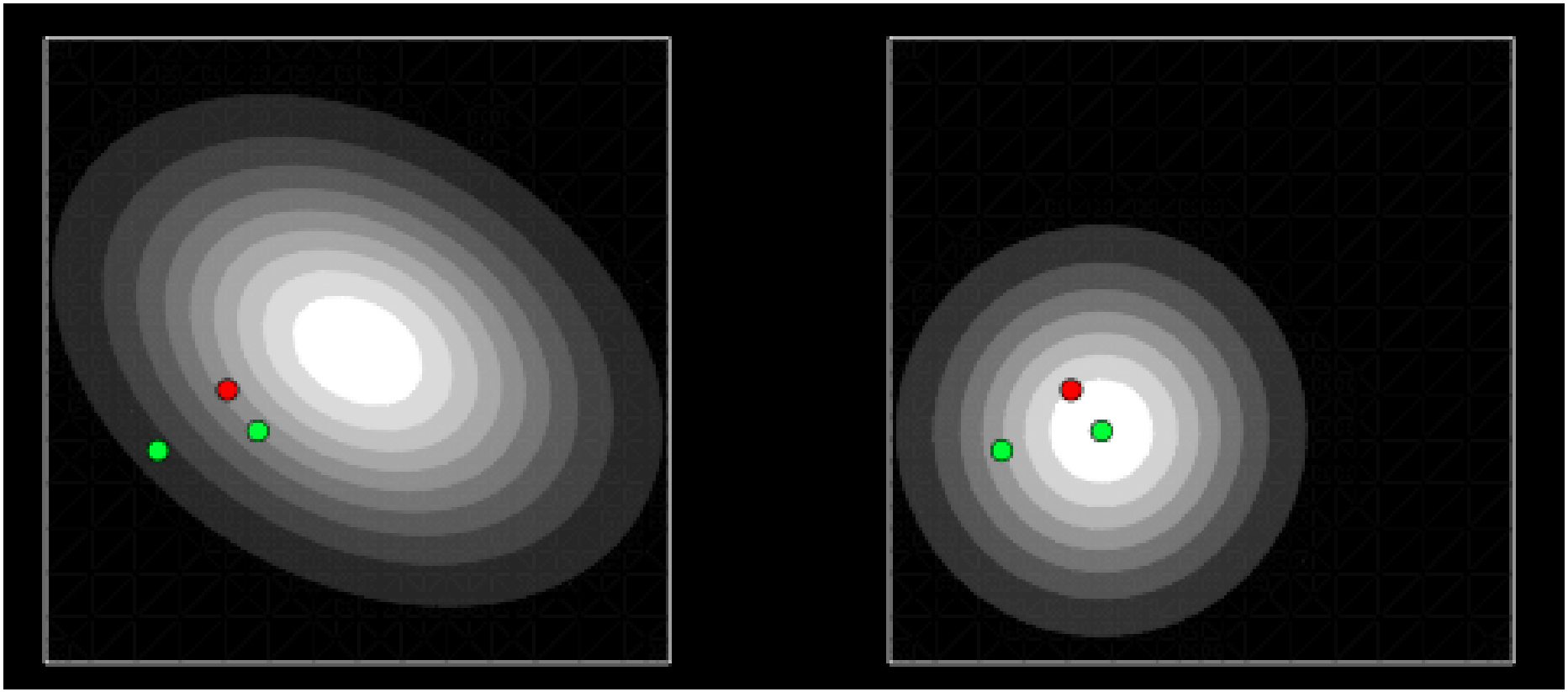

multiplicity + 1

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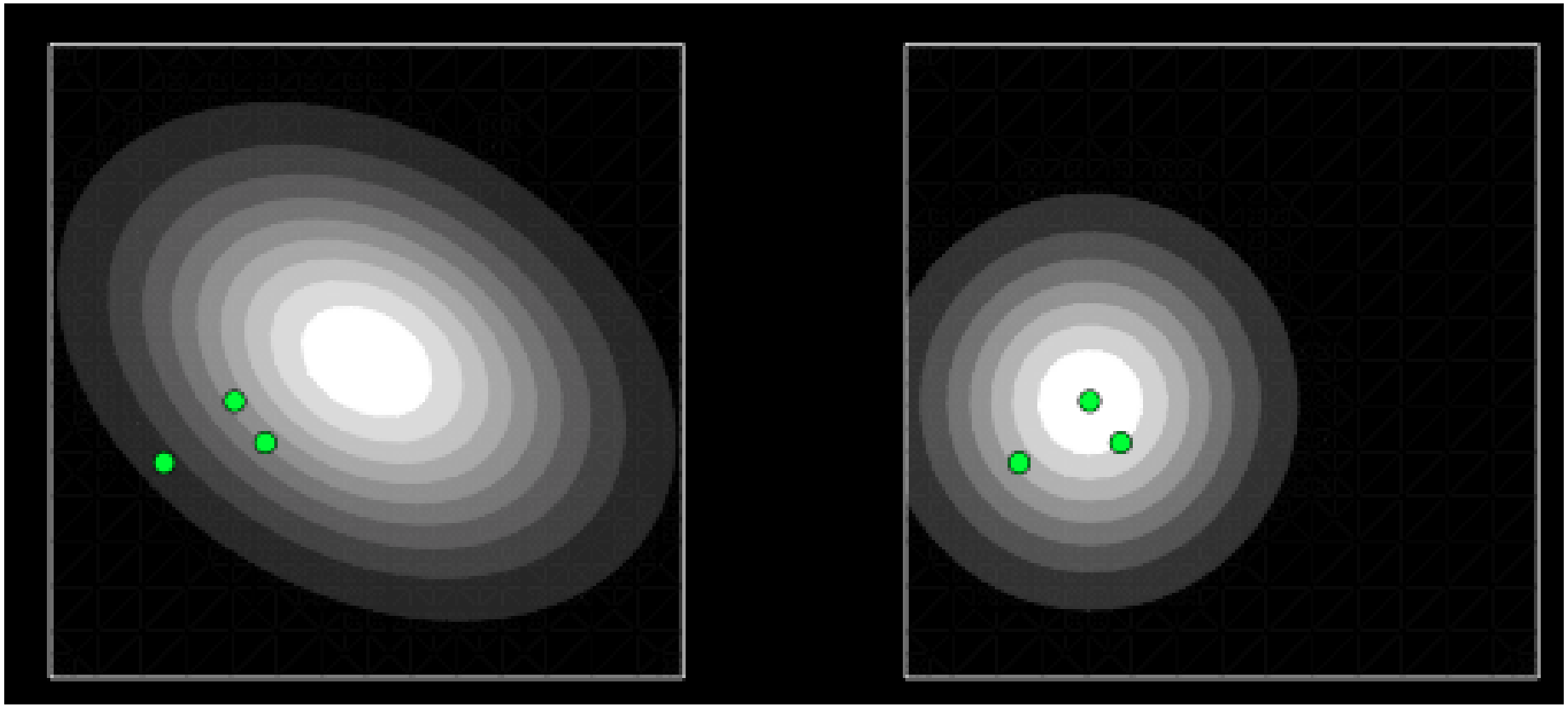

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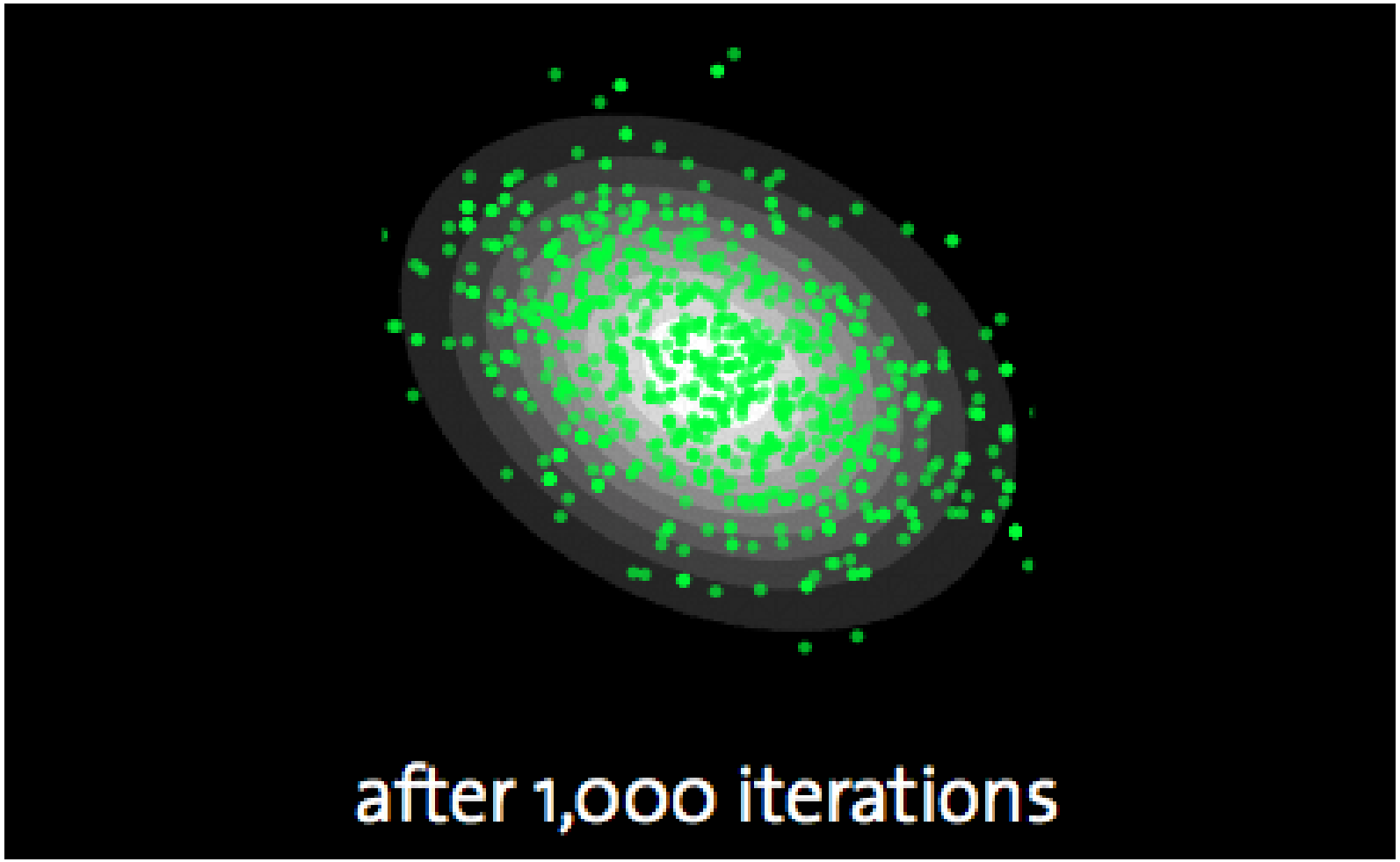
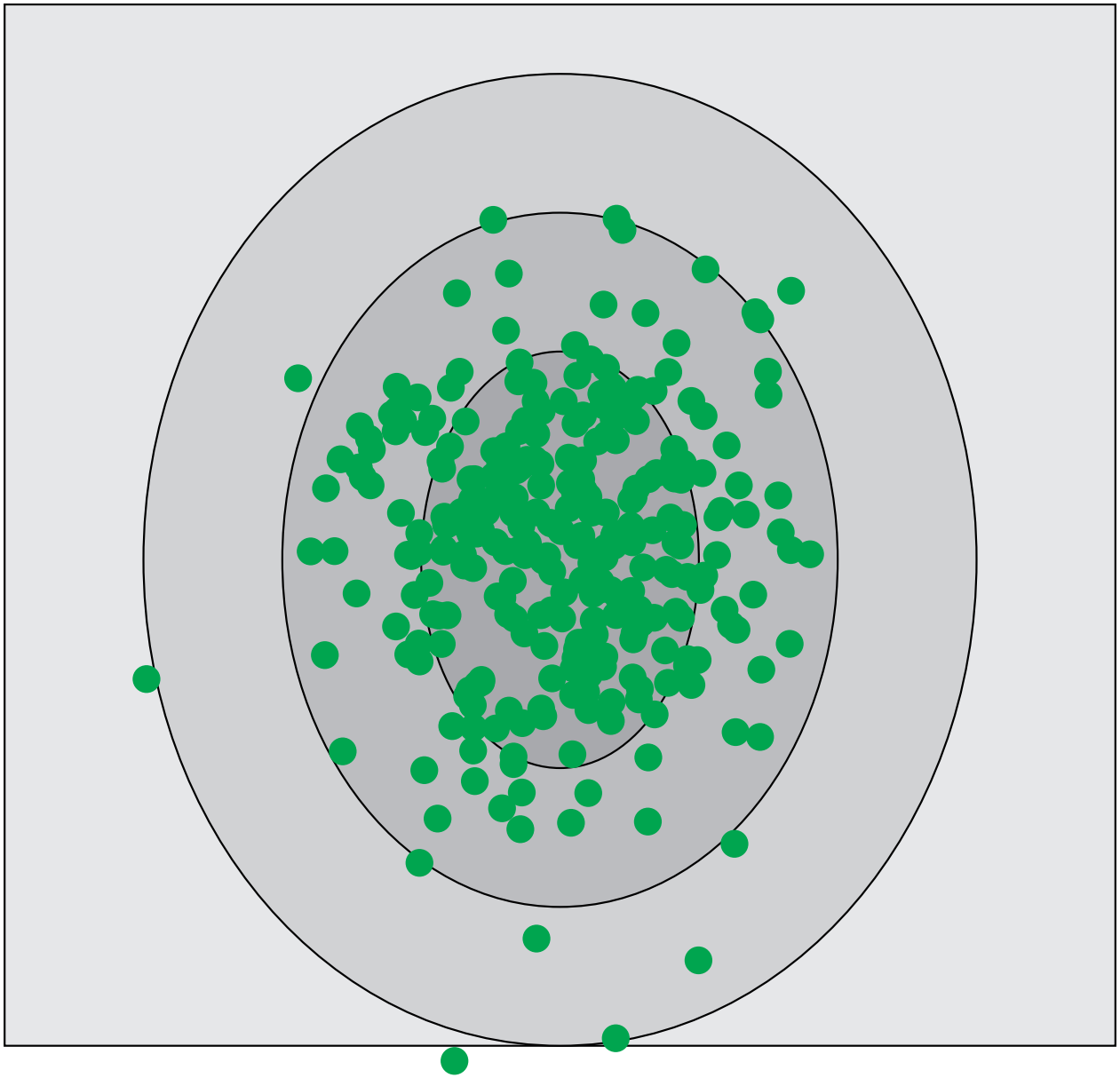


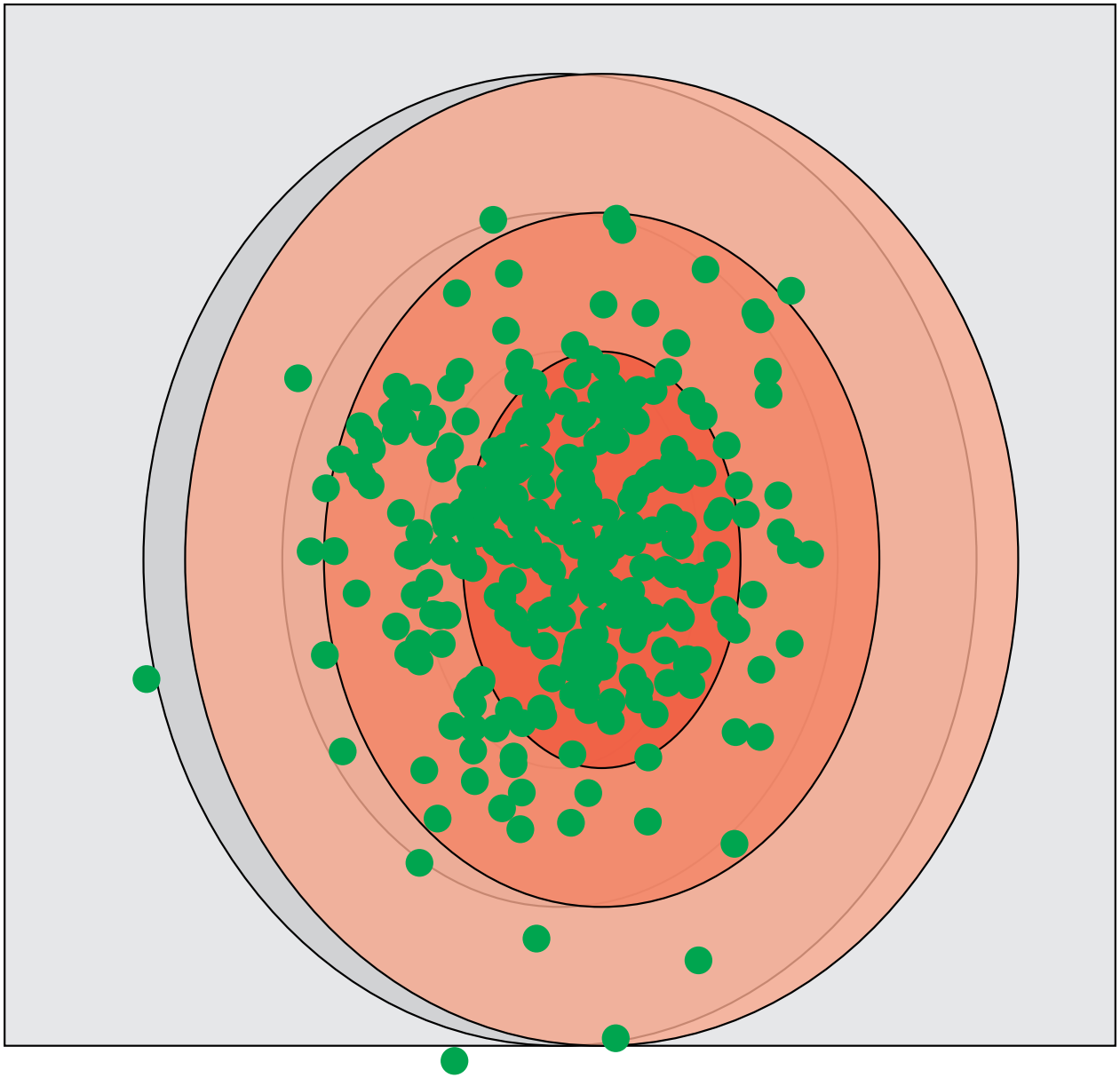
Importance Sampling

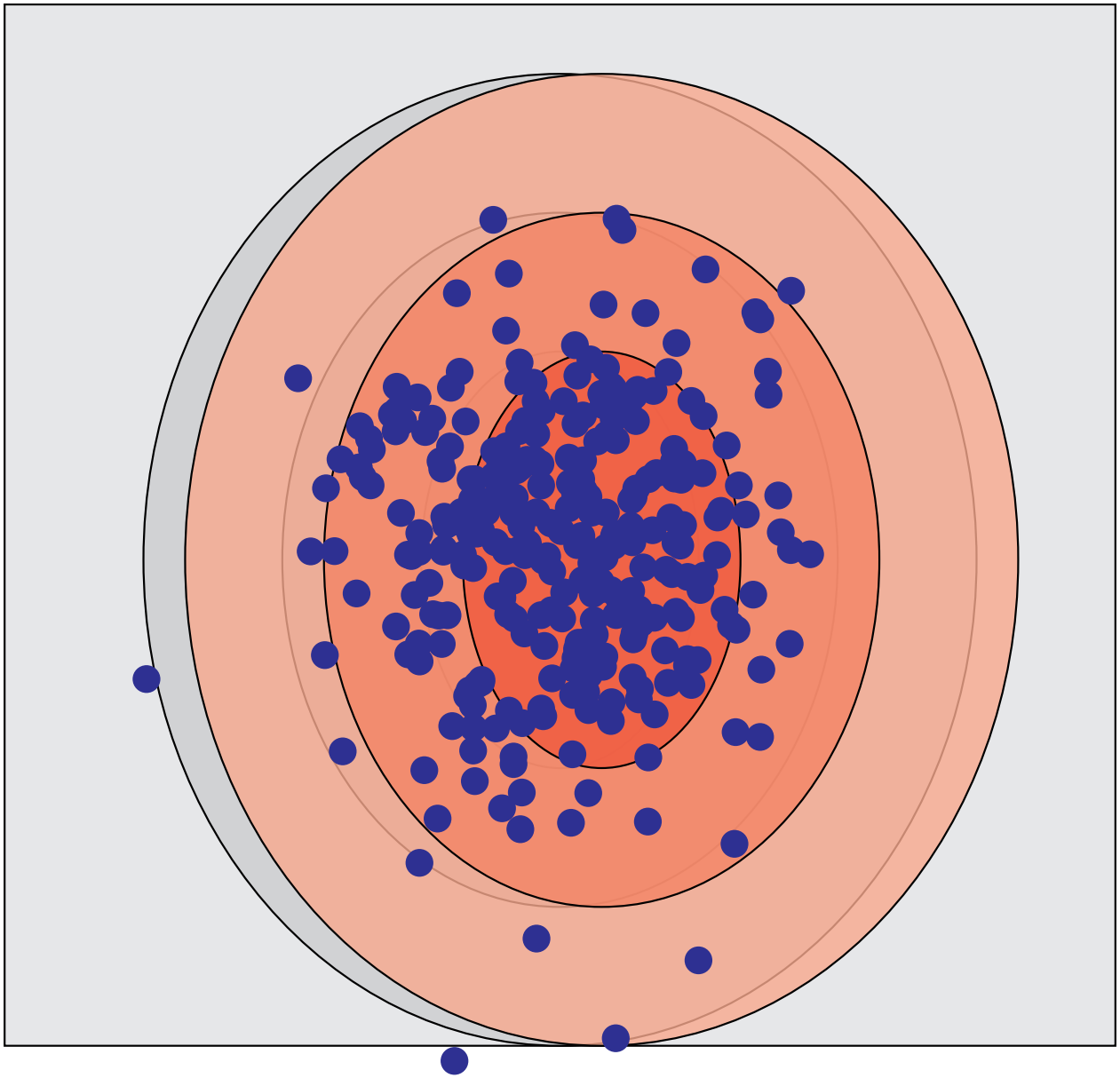
Not a real sampling

- start from an existing, **similar** distribution
- **add the new likelihood** at every already sampled point
- **write a new chain**, with the multiplicity changed:

$$\tilde{N} = N \frac{\tilde{\mathcal{L}}}{\mathcal{L}}$$







Benefits of Importance Sampling

When to use it?

- **Existing** run with a set of **slow** experiments
- Want to add one more experiment (**prior on** H_0)
- This should be very fast!