# Chapter IV — Overview of linear perturbation evolution in the standard, minimal $\Lambda$CDM model

## IV.1. Which scales? Which times? Which species?

Our goal: compute observables related to perturbations on large (cosmological) scales:

- spectrum of LSS (Large Scale Structure)
- CMB anisotropies
- weak lensing, etc.

*** on which scales?**

LSS → today, perturbations are growing; they are still described by linear perturbation theory for $\frac{k}{(a_0)} \leq 0.1\ h/\mathrm{Mpc} \Leftrightarrow \lambda \gtrsim 10\,h^{-1}\ \mathrm{Mpc}$

CMB → in decomposition in multipoles, CMB anisotropies observable for $1 \leq \ell \leq 2000$ (roughly)

above ~2000, signal supressed (Silk damping) and overseeded by foregrounds.

anisotropy associated to $\ell$ corresponds roughly to a scale on the last scattering surface seen under an angle $\theta = \frac{\pi}{\ell}$ radians.

We can <u>roughly estimate</u> wich comoving scale corresponds to a given $\ell$ in the following way:

⤳ $\ell=1$ (dipole) corresponds to the diameter of the observable Universe;
i.e. to comoving scale $k_{min}$ such that today

$$\lambda_{max}(t_0) = a(t_0)\frac{2\pi}{k_{min}} = 2R_H(t_0) = 6000\, h^{-1}\,\mathrm{Mpc}$$

⤳ $\ell = 2000$ corresponds to an angle 2000 times smaller on the last scattering surface, i.e. to a comoving scale <u>roughly</u> 2000 times smaller:

$$\lambda_{min}(t_0) = a(t_0)\frac{2\pi}{k_{max}} = \frac{6000}{2000}\, h^{-1}\,\mathrm{Mpc} = 3h^{-1}\,\mathrm{Mpc}$$

Using $a_0=1$ this gives $\begin{cases} k_{max} = 2\,h\,\mathrm{Mpc}^{-1} \\ k_{min} = 10^{-3}\,h\,\mathrm{Mpc}^{-1} \end{cases}$

This defines the range $[k_{min}, k_{max}]$ in which we want to understand the evolution of perturbations.

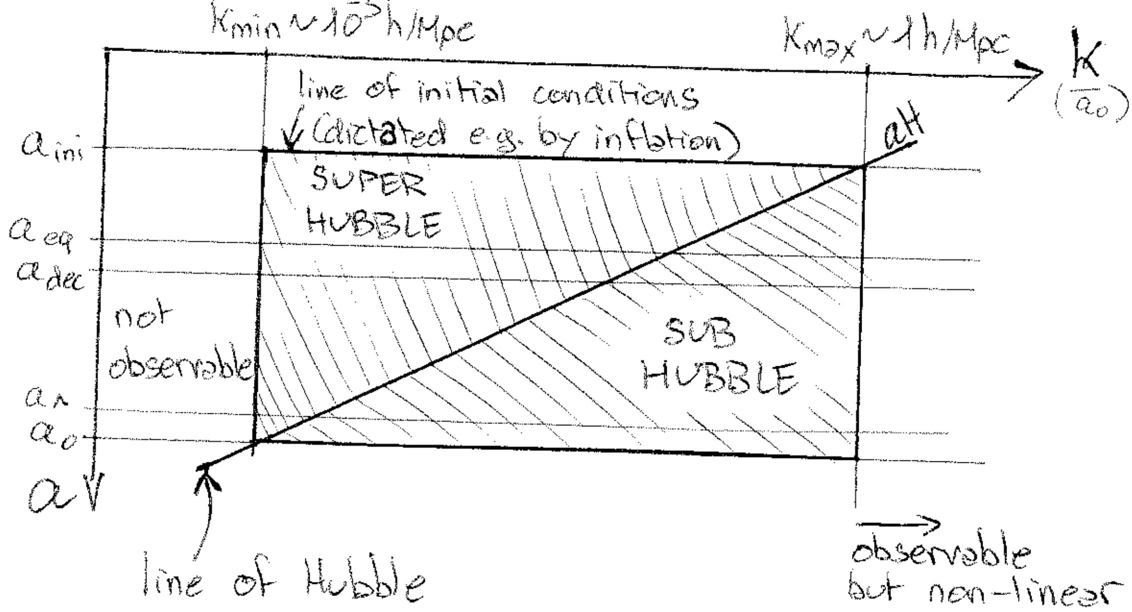
* <u>at which time ?</u>

We will see that above Hubble radius ($k \ll aH$) the perturbation evolution is trivial. So we are only interested in period starting when comoving scale $k_{min}$ approached the Hubble scale: this time

$t_i$ can be found by solving for $k_{min} = a(t_i) H(t_i)$.
Result: $t_i$ is such that $a_{ini} \sim \frac{a_{eq}}{100}$ ← scale factor at matter/radiation equality
(see exercises).

So $z_{ini} \sim 100\, z_{eq} \sim 10^5 - 10^6$

In summary we need to solve the equations inside the rectangle below:




$\left(aH = \frac{da}{dt} = \frac{d\ln a}{d\tau}\right.$ decreases after inflation$\left.\right)$

## * which species?

Since we are only intrerested in $z \lesssim 10^6$, we will only consider (at least in minimal $\Lambda$CDM):

- relativistic $\gamma$'s
- non-relativistic baryons "b" } coupled until $a_{dec}$
- non-relativistic CDM "c"
- relativistic neutrinos "$\nu$"