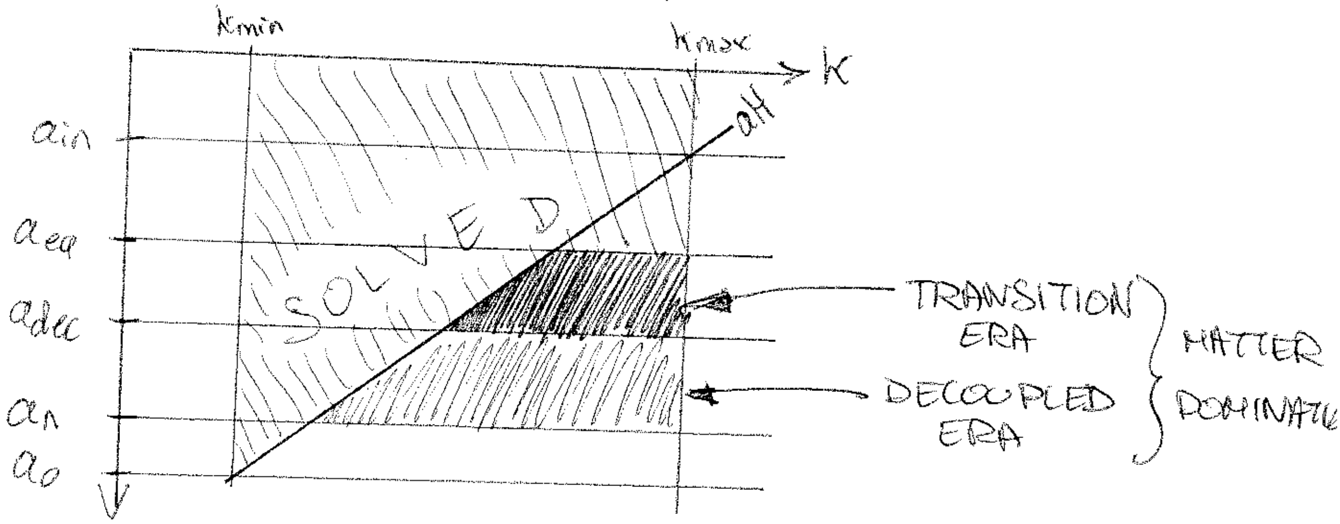
IV. 5. Sub-Hubble evolution during MD

MD can be decomposed in two parts: a transition epoch (between equality and photon decoupling), and a second epoch during which baryons and γs are decoupled.



The transition era is difficult to model analytically because no approximation is really good:

→ all densities of same order of magnitude

→ b-γ not highly coupled; c_s^2 in b+γ "fluid" decreases.

The decoupled era is very easy to model: b, CDM are collisionless; and we don't care anymore about photons...

In IV.5.A. we study the "easy" region; in IV.5.B. we come back to a qualitative description of the transition.

IV.S.A. Solution after τ_{dec}:

For $i=b$ or $i=c$:
$$\begin{cases} \delta_i' = \theta_i + 3\phi' \\ \theta_i' = -\frac{a'}{a}\theta_i - k^2\phi \end{cases}$$
(decoupled non-relativistic)

Einstein (III): $\left(2\frac{a''}{a} - \left(\frac{a'}{a}\right)^2\right)\phi + \frac{a'}{a}3\phi' + \phi'' = 4\pi G a^2 \delta\rho$

with $\delta\rho \simeq \delta\rho_b + \delta\rho_c \simeq 0$.

Using $a \propto t^{2/3} \Rightarrow a \propto \tau^2$, this gives:

$$\frac{6}{\tau}\phi' + \phi'' = 0$$

Solutions: $\phi(\vec{k},\tau) = D_1(\vec{k}) + D_2(\vec{k})(k\tau)^{-5}$

(valid inside and outside the Hubble radius).

Solution for δ_b and δ_c:

Continuity + Euler $\Rightarrow \delta_i'' + \frac{a'}{a}\delta_i' = -k^2\phi + \frac{3}{a}(a\phi')'$

As soon as the decaying mode is negligible ($\phi \simeq D_1$), the source term reads: $\delta_i'' + \frac{a'}{a}\delta_i' = -k^2\phi$ = constant

So $\delta_i'' + \frac{2}{\tau}\delta_i' = -k^2\phi$

Solution of homogeneous equation: $\delta_i = \mathrm{cst}$ or $\delta_i \propto \tau^{-1}$

Particular solution: $\delta_i = -\left(2 + \frac{k^2\tau^2}{6}\right)\phi$: wins over above solution

So $\delta_i \xrightarrow[k\tau \gg 1]{} -\frac{k^2\tau^2}{6}\phi = -\frac{k^2\tau^2}{6}D_1$

This also proves that $\delta_b \longrightarrow \delta_c$: normal since b and CDM fall in same potential wells.

Note that Poisson satisfied inside Hubble radius (ie for $k\tau \gg 1$):

$$\frac{\Delta}{a^2}\phi = 4\pi G\,\delta\rho_{tot} \Leftrightarrow -k^2\phi = 4\pi G a^2 \rho_{tot}\,\delta_{tot}$$

$$\Leftrightarrow -k^2\phi = \frac{3}{2}\left(\frac{a'}{a}\right)^2 \delta_{tot} = \frac{6}{\tau^2}\delta_{tot}$$

$$\Leftrightarrow \delta_{tot} = -\frac{k^2\tau^2}{6}\phi$$

with $\delta_{tot} = \frac{\Omega_b\delta_b + \Omega_c\delta_c}{\Omega_b+\Omega_c} = \delta_b = \delta_c$ when they converge towards each other.

Non-intuitive result: gravitational infall of matter corresponds to $\phi = cte$ instead of $\phi \nearrow$! This is due to the universe expansion:

$\delta_{tot} \sim \tau^2 \sim a$ grows (gravitational collapse)

$\rho_{tot} \sim a^{-3}$ (dilution)

$\Big\}\ \delta\rho_{tot} \sim a^{-2}$

... but physical Laplacian $\frac{\Delta}{a^2}$ goes like a^{-2} also (streching of distances) ...

so $\phi = cst$!

<u>Remark</u>: now CDM perturbations grow faster than during RD (like a instead of $\ln(a)$), because ϕ reacts to cdm, not to γ!

IV.5.B. Transition epoch: between τ_{eq} and τ_{dec}

Analytical solutions can be found in this regime, but they are very involved. Here we only provide a qualitative description.

Simplifying limit $\Omega_c \gg \Omega_b$:

[*] In this limit, after τ_{eq}, $\rho_{tot} \simeq \rho_c$ and $\delta_{tot} \simeq \delta_c$

[*] Poisson implies that ϕ evolves to value imposed by δ_c: $\phi \longrightarrow -\frac{a^2}{k^2} 4\pi G \rho_c \delta_c = -\frac{6}{k^2 \tau_{eq}^2} \delta_c$

[*] as soon as (p_γ, ρ_γ) is negligible, we know that ϕ must freeze-out with some value $D_1(\vec{k})$. So, very soon after equality, $\phi = D_1 \simeq -\frac{6}{k^2 \tau_{eq}^2} \delta_c(\vec{k}, \tau_{eq})$

Since we computed $\delta_c(\vec{k}, \tau_{eq}) \simeq -C_1(\vec{k}) \ln(k\tau_{eq})$ in chapter IV.4., it appears that the freeze-out value of $\phi(\vec{k}, \tau)$ for τ during matter domination, and k such that the mode entered inside R_H during RD ($k \geq a_{eq} H_{eq}$) is:

$$\boxed{\phi(\vec{k}, \tau) = +\frac{6}{k^2 \tau_{eq}^2} \ln(k\tau_{eq})\, C_1(\vec{k})}$$

[*] CDM experiences gravitational clustering (like after τ_{dec}):

$$\delta_c = -\frac{k^2 \tau^2}{6} \phi = -\left(\frac{\tau}{\tau_{eq}}\right)^2 \ln(k\tau_{eq})\, C_1(\vec{k})$$

- baryons are still coupled to photons (although not perfectly), so $\delta_b \simeq \frac{3}{4}\delta_\gamma$. Because c_s^2 is decreasing in the b+γ fluid, they both experience <u>damped oscillations</u> until decoupling. Later on, we know that $\delta_b \to \delta_c \to -\frac{k^2\tau^2}{6}\phi = -\left(\frac{\tau}{\tau_{eq}}\right)^2 h_c(k\tau_{eq})\, C_\ell(\vec{k})$, and we do not care about photons any more.

<u>Simplifying limit $\Omega_b \gg \Omega_c$:</u>

- In this limit, after τ_{eq}, $\rho_{tot} \simeq \rho_b$ and $\delta_{tot} \simeq \delta_b$
- Poisson implies that ϕ jumps to the value imposed by baryons, and then, keep tracking δ_b:

$$\phi \longrightarrow -\frac{a^2}{k^2} 4\pi G \rho_b \delta_b = -\frac{6}{k^2\tau^2}\delta_b$$

- Meanwhile, the b+γ fluid in which c_s^2 decreases experiences <u>damped oscillations</u> until decoupling.
- At decoupling, ϕ freeze-out to the value

$$\phi \to -\frac{6}{k^2\tau_{dec}^2}\delta_b(\vec{k}, \tau_{dec})$$

which reflects the last damped oscillations of δ_b.

- After decoupling, δ_b grows like $-\frac{k^2\tau^2}{6}\phi$ (with ϕ equal to the above value).

Summary: during MD, $\delta_m = \delta_b = \delta_c$ always grow like $-\frac{k^2\tau^2}{6}\phi$ where ϕ is constant at least after τ_{dec}. What changes is the freeze-out value of ϕ for modes which entered inside R_H during RD:

→ if $\Omega_b \gg \Omega_c$: $\phi^{\text{freeze-out}}$ imposed by CDM at equality: $\phi = \frac{6}{k^2\tau_{eq}^2} f_{tc}(k\tau_{eq})\, C_\phi(\vec{k})$

→ if $\Omega_b \ll \Omega_c$: $\phi^{\text{freeze-out}}$ imposed by baryons at decoupling: with respect to above result, ϕ is suppressed and carries oscillatory patterns due to the last damped acoustic oscillations.

→ realistic situations ($\Omega_b \sim \Omega_c$) interpolate between these two limits.

Note: for modes entering R_H, after τ_{dec}, the evolution is trivial: as derived in IV.5.A and IV.3.(

$$\phi(\vec{k}, \tau_{MD}) = D_\phi(\vec{k}) = \frac{9}{10}\phi(\vec{k}, \tau_{RD})$$

$$\delta_c(\vec{k}, \tau_{MD}) = \delta_b(\vec{k}, \tau_{MD}) = -\left(2 + \frac{k^2\tau^2}{6}\right)\phi(\vec{k}, \tau_{MD})$$