V.2 Equation of evolution for baryons

Baryons = non-relativistic, equivalent to pressureless fluid.
Described by (δ_b, θ_b) only, with continuity + Euler equations including coupling term (with photons, through electrons).
Proper way to obtain this term: write Boltzmann equation, incorporation Thomson scattering term btw γ and e^-, and using tight-coupling limit for Coulomb scattering

$\Rightarrow \vec{v}_b = \vec{v}_{e^-} \Rightarrow \theta_b = \theta_e$

Result:

$$\boxed{\begin{cases} \delta_b' = \theta_b + 3\psi' \\ \theta_b' = -\frac{a'}{a}\theta_b - k^2\phi + \frac{4}{3}\frac{\bar{\rho}_\gamma}{\bar{\rho}_b}\underbrace{a n_e \sigma_T}_{-\tau'}(\theta_\gamma - \theta_b) \end{cases}}$$

Remark: coupling term would vanish in limit $\rho_b \to \infty$.
Interpretation: if $m_b \to \infty$, impossible to move baryons, and hence to move electrons: then baryons unaffected by Thomson scattering of γ over e^-.

Summary: full evolution of cosmological perturbations in Fourier space can be infered from:

* continuity + Euler for baryons: $\begin{cases} \delta_b' = \dots \\ \theta_b' = \dots \end{cases}$

* Boltzmann for photons: $\begin{cases} \delta_\gamma' = \dots \\ \theta_\gamma' = \dots \\ \sigma_\gamma' = \dots \\ \Theta_{\ell \geq 3}' = \dots \end{cases}$

* continuity + Euler for CDM: $\begin{cases} \delta_c' = \dots \\ \theta_c' = \dots \end{cases}$

* constraint equations: Einstein $\to \phi, \psi = \dots$